REPORT No. 638

THE INFLUENCE OF LATERAL STABILITY ON DISTURBED MOTIONS OF AN AIRPLANE WITH SPECIAL REFERENCE TO THE MOTIONS PRODUCED BY GUSTS

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SUMMARY

Disturbed lateral motions have been calculated for a hypothetical small airplane with various modifications of fin area and dihedral setting. Special combinations of disturbing factors to simulate gusts are considered and the influence of lateral stability on the motions is discussed.

The modifications of the airplane include changes of dihedral from 0° to 10° and changes of the weathercock stability from zero to $C_{n_\beta}=0.137$ (the equivalent of a fin as large as 10 percent of the wing area). The positions of the modified airplanes on the lateral-stability charts are shown.

Fin area and wing dihedral were found to be of primary importance in side gusts. It was found that the rolling action of the wing with as much as 5° dihedral was distinctly unfavorable, especially when the weathercock stability was small. It is pointed out that the greatest susceptibility to lateral disturbances lies in the inherent damping and coupling moments developed by the wing.

INTRODUCTION

Inherent stability, as defined in mathematical treatment, must be considered only one of several essential flying qualities of an airplane. Other important qualities belonging in this category are steadiness in rough air and responsiveness to control. Although the different flying qualities depend largely on the same governing factors, they may not call for similar proportionings of the factors. It is known, for instance, that the requirements for stability and control may conflict.

What is ultimately desired, or course, is a definite understanding of the individual requirements for stability, control, and steadiness in rough air. Most of the earlier work has been devoted primarily to the study of stability alone. A noteworthy early work on the effects of gusts is that of Wilson (reference 1). More recently the results of an investigation dealing with the effects of different degrees of stability on the motions following assumed initial conditions have been published (reference 2). The purpose of the present work is to study the amplitudes of the motions set up by gusts or other disturbances, particularly insofar as these motions are affected by the lateral-stability characteristics. It is hoped that the study will be useful in indicating combinations of stability characteristics that result in good riding qualities.

The mathematical treatment employed is, in principle, an extension of that used by Wilson and other early writers. The methods of calculation are, however, more concise and the development is not restricted to special types of gust. The operational method of resolving the effects of disturbances was used. (See reference 3.)

According to the theory, the motion caused by any random variation or sequence of the disturbing factors may be built up by superposing the effects of abrupt unit increases of the disturbance, which corresponds, in the case of gusts, to the effects of elementary sharp-edge cross-currents. Thus the effects of random gusts can be largely visualized in the effect of a unit sharp-edge gust.

STABILITY FACTORS ASSUMED

The chief differences of lateral stability considered were assumed to be brought about by changing the fin area and dihedral of a hypothetical small monoplane. Differences in other proportions of most airplanes of conventional form have only secondary effects (in unstalled flight) and, furthermore, are not usually dictated by considerations of stability. The exact arrangement of the hypothetical airplane, such as the vertical disposition of the wing with respect to the fuselage, may be taken as indefinite. Differences of arrangement can, of course, have large secondary influences on the action of the fin or dihedral, which are usually attributed to aerodynamic interference. It is reasonable to assume that the effects of such interference will be similar to the effects of actual changes in the size of the fin or the amount of dihedral.

The airplane assumed in the calculations is a small 1,600-pound monoplane having rectangular wings with rounded tips. The other proportions, including the radii of gyration about various axes, the tail length, etc., are based on average values of these quantities for a number of conventional machines. The stability derivatives and other characteristics of the airplane are essentially the same as those used in reference 4, except for the differences of fin area and dihedral, and apply to power-off flight. The axes and symbols employed throughout are given in detail in reference 4. Additional symbols that occur in this report are given in the following list:

X, Y, and Z, axes fixed in the airplane so that X points into the relative wind in steady flight. (See report cover.)

U_0, steady-flight velocity.

$\beta=\tan^{-1} v/U_0$.

Derivatives (see report cover for formulation of coefficients):

$C_{Y_\beta}=\dfrac{\partial C_Y}{\partial\beta}$, side force due to sideslip.

$C_{l_\beta}=\dfrac{\partial C_l}{\partial\beta}$, rolling moment due to sideslip.

$C_{n_\beta}=\dfrac{\partial C_n}{\partial\beta}$, yawing moment due to sideslip.

$C_{l_p}=\dfrac{\partial C_l}{\partial\frac{pb}{2U_0}}$, rolling moment due to rolling.

$C_{n_p}=\dfrac{\partial C_n}{\partial\frac{pb}{2U_0}}$, yawing moment due to rolling.

$C_{l_r}=\dfrac{\partial C_l}{\partial\frac{rb}{2U_0}}$, rolling moment due to yawing.

$C_{n_r}=\dfrac{\partial C_n}{\partial\frac{rb}{2U_0}}$, yawing moment due to yawing.

$L_v=\dfrac{1}{mk_X^2}\dfrac{\partial L}{\partial v}$.

$N_v=\dfrac{1}{mk_Z^2}\dfrac{\partial N}{\partial v}$.

It was found convenient to designate the five cases of modification by symbols representing the different front views of the airplane. Table I gives the stability coefficients assumed in each case for flight at three different lift coefficients.

TABLE I.—ASSUMED STABILITY COEFFICIENTS

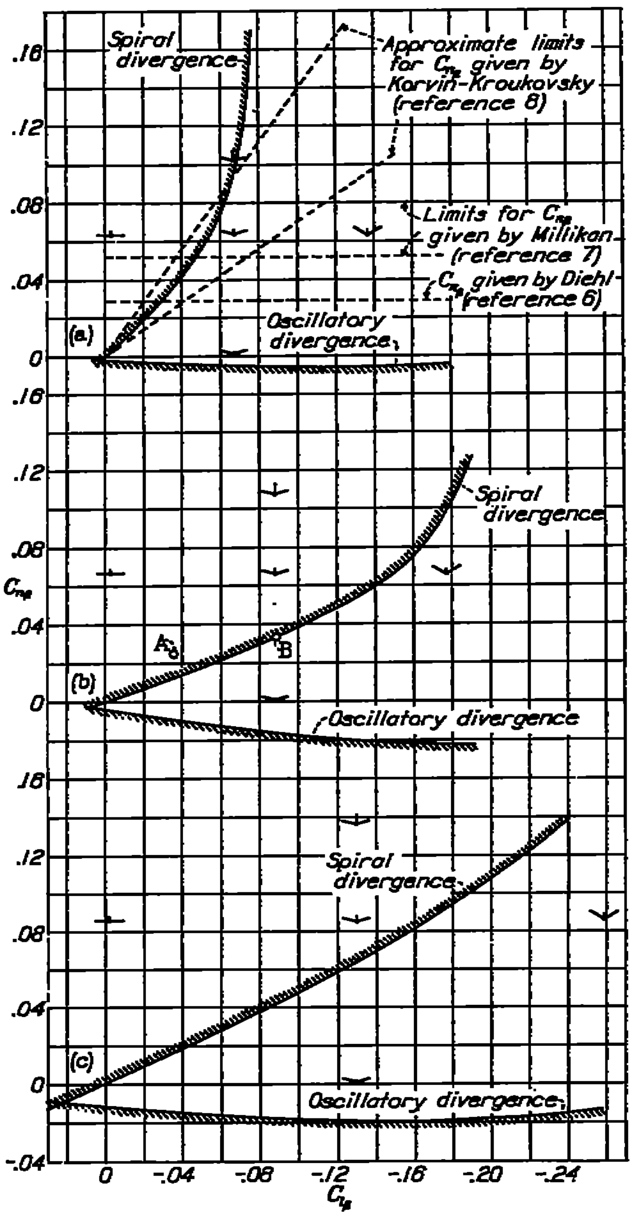
(b, 32 ft.; m, 49 slugs; S, 171 sq. ft.; k_X, 0.150b; k_Z, 0.183b)

Case	Ratio of vertical fin area to wing area	Dihedral angle (deg.)	C_L	U_0 (f. p. s.)	$C_{l_p}=\frac{\partial C_l}{\partial\frac{pb}{2U_0}}$	$C_{l_r}=\frac{\partial C_l}{\partial\frac{rb}{2U_0}}$	$C_{l_\beta}=\frac{\partial C_l}{\partial\beta}$	$C_{n_p}=\frac{\partial C_n}{\partial\frac{pb}{2U_0}}$	$C_{n_r}=\frac{\partial C_n}{\partial\frac{rb}{2U_0}}$	$C_{n_\beta}=\frac{\partial C_n}{\partial\beta}$	$C_{Y_\beta}=\frac{\partial C_Y}{\partial\beta}$
	0.04	5.0	0.35	150.0	−0.425	0.086	−0.067	−0.022	−0.076	0	−0.226
			1.00	88.5	−.420	.250	−.088	−.055	−.091	0	−.356
			1.80	66.0	−.442	.442	−.130	−.074	−.196	0	−.773
	.06	5.0	.35	150.0	−.425	.086	−.067	−.022	−.097	.064	−.274
			1.00	88.5	−.420	.250	−.088	−.055	−.109	.067	−.410
			1.80	66.0	−.442	.442	−.130	−.074	−.220	.095	−.808
	.19	5.0	.35	150.0	−.425	.086	−.067	−.022	−.130	.102	−.378
			1.00	88.5	−.420	.250	−.088	−.055	−.146	.108	−.404
			1.80	66.0	−.442	.442	−.130	−.074	−.246	.137	−.920
	.06	0	.35	150.0	−.425	.086	0	−.022	−.097	.064	−.274
			1.00	88.5	−.420	.250	0	−.055	−.109	.067	−.410
			1.80	66.0	−.442	.442	0	−.074	−.220	.086	−.808
	.06	10.0	.35	150.0	−.425	.086	−.137	−.022	−.097	.064	−.274
			1.00	88.5	−.420	.250	−.177	−.055	−.109	.067	−.410
			1.80	66.0	−.442	.442	−.269	−.074	−.220	.055	−.808

These coefficients were estimated from the outward characteristics of the airplane by methods described in reference 4. The derivatives C_{n_β}, C_{n_r}, and C_{Y_β} (corresponding to the yawing moments in sideslip and in yawing and to the side force in sideslip) were assumed to be affected by the changes of fin area. Only the derivative C_{l_β} (corresponding to the rolling moment in sideslip) was assumed to be affected by changes of dihedral. The effect of dihedral on the lateral force in sideslip was neglected inasmuch as it was found that a compensating error was introduced by the absence of the side force due to rolling in the equations of motion. Another omission is the small adverse effect of dihedral angle on the weathercock-stability factor C_{n_β}. This effect is small, particularly in view of the wide variation of C_{n_β} assumed. At a lift coefficient of 1.8, representing low-speed flight, a full-span flap was assumed. Tests show that the effect of such a flap is to increase the weathercock-stability factor somewhat for the wing alone. In practice, the flap might interfere with the air flow over the fin so that the increase of C_{n_β} assumed in this condition would not be realized.

Figure 1 shows the positions of the modified airplanes on the lateral-stability diagrams. These diagrams are essentially similar to those given in reference 5 except that a simultaneous increase of C_{n_r} with C_{n_β} was assumed to show directly the effect of increasing the fin area.

The value recommended by Diehl (reference 6) for C_{n_β} works out to about 0.03 for the wing loading assumed here. Limits mentioned by Millikan (reference 7) correspond to $0.08>C_{n_\beta}>0.05$. Nearly all designers are familiar with the limits of L_v/N_v for satisfactory lateral stability given by Korvin-Kroukovsky (reference 8). Figure 1 (a) shows these limits in terms of C_{l_β} and C_{n_β}. It should be mentioned that Korvin-Kroukovsky's formulas are more suited to the empirical-statistical analysis in which they were employed than to the determination of absolute values of C_{n_β}/C_{l_β} for this stability chart. In most cases, wind-tunnel tests show values of C_{n_β} smaller than those predicted so that



(a) $C_L=0.35$; $U_0=150$ f. p. s.
(b) $C_L=1.0$; $U_0=88.5$ f. p. s.
(c) $C_L=1.8$; $U_0=66$ f. p. s.

FIGURE 1.—Stability chart showing positions of the assumed airplanes.

the specified range, if given in wind-tunnel values, would probably fall somewhat lower than indicated in figure 1.

The value $C_{n_\beta}=0$ does not, of course, correspond to an airplane with no vertical tail surface. Experience has shown that the unstable yawing moment of a large well-streamlined fuselage may entirely offset the stabilizing action of a fair-size fin. This occurrence is naturally more probable if the fin area is originally small; hence the smallest area likely to be used in a modern design (4 percent of the wing area) was chosen to represent the condition.

It is, in general, difficult to predict the values of either C_{n_β} or C_{l_β} for a given design. It will be realized that the corresponding values of fin area and dihedral, as referred to in this report, apply only under certain idealized conditions and are employed primarily as a matter of convenience in fixing ideas on the problem. Reference 9 gives a summary of test values of C_{n_β}, including a discussion of pertinent factors and drawings of the models tested. The data included in that paper should aid the designer in judging the weathercock stability.

INFLUENCE OF LATERAL STABILITY ON MOTIONS DUE TO ARBITRARY DISTURBANCES

GENERAL DESCRIPTION OF LATERAL MOTIONS

The equations of lateral stability generally show two real roots together with one conjugate complex pair, indicating three "modes" of motion. Different disturbances will result in motions compounded of these three modes in different proportions.

Table II lists the roots, or stability indices, for the various cases considered. The first mode (corresponding to the root λ_1), represents primarily the heavy damping of any movement involving rolling of the wings relative to the air. At normal flight speeds, this damping is such that the wings are in a large measure constrained against such relative movement normal to their chords.

TABLE II.—STABILITY INDICES, RATES OF DAMPING, AND PERIODS

[$\lambda_3=a+bi$]

Case	C_L	λ_1	$\frac{-0.69}{\lambda_1}$ (a)	λ_2	$\frac{(0.095 \text{ or } -0.105)}{\lambda_2}$ (b)	a	$\frac{-0.69}{a}$	b	$\frac{\pi}{2b}$ (c)
	0.35	−5.477	0.126	−0.4190	[d] 1.645	−0.1720	4.01	0.706	2.22
	1.0	−3.255	.212	−.2974	[d] 2.32	−.1386	4.98	.8241	1.907
	1.8	−2.533	.273	−.6080	[d] 1.13	−.1794	3.85	.8568	1.83
	.35	−5.472	.126	−.0070	15.0	−.495	1.39	2.45	.641
	1.0	−3.280	.210	.0615	1.55	−.3637	1.90	1.678	.937
	1.8	−2.633	.262	.0624	1.53	−.5247	1.315	1.546	1.02
	.35	−5.466	.126	0	∞	−.680	1.01	2.08	.818
	1.0	−3.294	.209	.0638	1.133	−.4797	1.44	2.013	.780
	1.8	−2.670	.258	.1247	.706	−.6075	1.14	1.829	.858
	.35	−5.369	.129	.0420	2.27	−.571	1.21	2.35	.668
	1.0	−3.070	.225	.1770	.537	−.529	1.30	1.519	1.034
	1.8	−2.320	.297	.2900	.328	−.795	.868	1.415	1.110
	.35	−5.573	.124	−.0504	2.08	−.423	1.63	2.55	.616
	1.0	−3.460	.199	−.0184	5.71	−.235	2.94	1.826	.860
	1.8	−2.872	.240	−.1045	1.005	−.3219	2.14	1.697	.925

[a] The value $-0.69/\lambda$ represents time to diminish by ½.
[b] Use $-0.105/\lambda_2$ for time to diminish by 1/10 when λ_2 is negative; use $0.095/\lambda_2$ for time to increase by 1/10 when λ_2 is positive.
[c] Quarter period.
[d] For this case, $-0.69/\lambda_2$ has been used.

The second mode distinguishable in the lateral motions (corresponding to λ_2) is a practically continuous turning motion, which may either converge or diverge. Normal stability of this mode represents the slow natural recovery from a banked turn. The rate of increase or decrease of the turning motion is slow, primarily on account of the insensitiveness of the airplane to displacement in bank and the strong resistance to rolling motion. The slow spiral always occurs with inward sideslip.

The third mode (λ_3) is the familiar oscillation, consisting usually of a yawing and sideslipping motion. Such rolling as occurs in the oscillation is determined by the tendency of the wings to follow a path outlined by the dihedral in front view. The wing, when sideslipping, tends strongly to roll in a way involving the least angle-of-attack change along the span. Thus the oscillations involve a "weathercock" motion combined with a rolling nearly in phase with the sideslip.

With fairly large fin area, the oscillations are rapid and are quickly damped. Under most conditions the amplitude is small. As the fin area is reduced, however, the period becomes slower and, with normal dihedral, the oscillation takes on the character of a swinging in bank and sideslip under the action of gravity. The point of instability is reached when the oscillation degenerates to an almost pure rolling and sideslipping motion, so that the damping derivative in yawing, C_{n_r}, has little effect on the occurrence of undamped oscillations. As has been shown (reference 4), unstable oscillations can readily occur if the airplane is constrained in yawing.

Although both the oscillation and the slow mode of convergence are largely governed by the fin area and the dihedral, the rapid convergence λ_1 is practically independent of either of these factors. The damping comes, of course, from the wings and is an inherent characteristic of conventional airplanes. This damping is excessive and is undesirable, since it indicates great sensitiveness to rolling gusts or to vertical gusts with a gradient along the wing span. The damping of rolling can be reduced by increasing the lateral moment of inertia, but the possible improvement appears to be small.

CALCULATED MOTIONS

The equations of motion of the airplane form a linear system so that the effects of disturbances can be compounded by addition. Thus, if any sequence of application of forces or couples to the airplane is given, it is possible to compute the resultant theoretical motion at any instant by addition, or integration, of separate effects. The impressed forces or couples may be due to control manipulation or to gusts, alone or in combination.

The foregoing statement refers to a resolution of the impressed disturbances along the axes fixed in the airplane. The disturbances are assumed to take on preassigned values independent of the movements of the airplane. With conventional control devices, the disturbances do remain practically independent of the motions. The orientation of the gusts is not dependent on the motion of the airplane, and deviations caused by such outside disturbances will introduce changes in the magnitudes of the disturbing factors themselves. For small displacements, these changes are of second order and are negligible. For large displacements in gusts it may, however, be necessary to carry out the calculation in several steps, altering the magnitude of the disturbance as the orientation of the relative wind changes.

The data needed for the computation of motion under any given set of conditions are the histories of motions following sudden unit disturbances. Computations of such unit motions were made during the course of the investigation reported herein and the results were used as the basis for the more complete calculations given later.

Specifically, the unit disturbance referred to is a force (Y) or a couple (L or N) having the value zero up to the time $t=0$ and maintaining a constant value thereafter. The magnitude is such as to cause a unit linear or angular acceleration of the airplane. According to well-known mathematical rules, the motions under such conditions are given by equations of the form

$$p_Y(t)=p_{Y0}+p_{Y1}e^{\lambda_1 t}+p_{Y2}e^{\lambda_2 t}+p_{Y3}e^{at}\cos b(t+t_{pY}) \quad (1)$$

where p_{Y0}, p_{Y1}, and t_{pY} are constants, calculated values of which are given in table III, and λ_1, λ_2, $a+ib$, and $a-ib$ are the roots, or stability indices. (See table II.) Three components of motion for each of three component disturbances are given. Thus, $p_Y(t)$ denotes the rolling velocity due to a unit side disturbance and $r_L(t)$ denotes the yawing velocity due to a unit rolling disturbance.

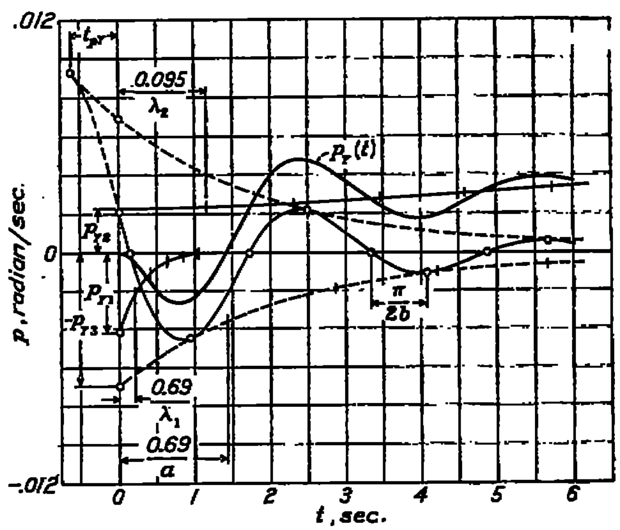


FIGURE 2.—Example showing use of data given in table III for plotting motions.
$p_Y(t)=p_{Y1}e^{\lambda_1 t}+p_{Y2}e^{\lambda_2 t}+p_{Y3}e^{at}\cos b(t+t_{pY})$

Plots of these equations were made with scarcely any additional computation. The procedure is illustrated by figure 2. First the coefficients, as given in

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TABLE III.—EQUATIONS OF MOTION FOR UNIT DISTURBANCES

[$v_L(t)=v_{L0}+v_{L1}e^{\lambda_1 t}+v_{L2}e^{\lambda_2 t}+v_{L3}e^{at}\cos b(t+t_{vL})$; etc.]

a—Due to side disturbance

Case	C_L	Sideslip v_{Y0}	v_{Y1}	v_{Y2}	v_{Y3}	t_{vY}	Rolling p_{Y0}	p_{Y1}	p_{Y2}	p_{Y3}	t_{pY}	Yawing r_{Y0}	r_{Y1}	r_{Y2}	r_{Y3}	t_{rY}
	0.35	0	−0.0041	0.5085	−1.6620	1.735	0	−0.00178	−0.00683	0.01688	1.600	0	−0.00007	0.00330	−0.00390	0.284
	1.0	0	−.0287	.5558	−1.3410	1.413	0	−.00374	−.00408	.01653	1.320	0	−.00041	.00605	−.00565	−.060
	1.8	0	−.067	.627	−1.416	1.360	0	−.00058	−.00497	.02360	1.235	0	−.00118	.00777	−.00668	.138
	.35	0	−.0033	.1390	−.3960	.500	0	−.00171	−.00021	.00398	.433	0	−.00005	.00381	−.00339	−.088
	1.0	0	−.021	.274	−.582	.659	0	−.00414	.00159	.00804	.743	0	−.00080	.00578	−.00388	−.175
	1.8	0	−.053	.446	−.565	.518	0	−.00734	.001509	.0121	.690	0	−.00060	.00075	−.0102	−.297
	.35	.1271	−.0027	0	−.3310	.891	−.00015	−.00169	0	.00332	.318	.00625	−.00040	0	−.00038	−.070
	1.0	0	−.0199	.2350	−.4570	.587	0	−.00433	.00294	.00698	.629	0	−.00023	.00908	−.00980	−.155
	1.8	0	−.048	.342	−.459	.479	0	−.00770	.00828	.0110	.629	0	−.00037	.00970	−.0108	−.238
	.35	0	−.0005	.1087	−.4200	.498	0	−.00036	.00128	−.00184	−.302	0	−.00001	.00635	−.00656	−.110
	1.0	0	−.0087	.3510	−.5040	.680	0	−.00153	.00450	−.00681	−.641	0	−.00010	.00654	−.00939	−.301
	1.8	0	−.044	.489	−.529	.402	0	−.00485	.00660	−.0106	−.993	0	−.00085	.00738	−.00928	−.509
	.35	0	−.0056	.1208	−.3430	.487	0	−.00305	−.00142	.00746	.408	0	−.00010	.00607	−.00600	−.069
	1.0	0	−.0278	.2080	−.5060	.660	0	−.00601	−.00044	.01236	.557	0	−.00041	.00846	−.00824	−.115
	1.8	0	−.057	.329	−.537	.612	0	−.00909	−.00226	.01755	.511	0	−.00075	.00996	−.00952	−.149

b—Due to rolling disturbance

Case	C_L	v_{L0}	v_{L1}	v_{L2}	v_{L3}	t_{vL}	p_{L0}	p_{L1}	p_{L2}	p_{L3}	t_{pL}	r_{L0}	r_{L1}	r_{L2}	r_{L3}	t_{rL}
	0.35	18.490	−0.401	2.985	−22.085	−0.419	0	−0.17002	−0.02753	0.22000	−0.552	0	−0.0074	0.01959	−0.04996	−1.870
	1.0	28.809	−1.7368	2.2288	−26.832	−.480	0	−.2787	−.0163	.3338	−.591	0	−.08029	.03409	−.11130	−1.832
	1.8	21.688	−3.452	5.385	−26.215	−.528	0	−.3341	−.0431	.4385	−.625	0	−.0576	.0572	−.1267	−1.744
	.35	114.008	−.838	−112.570	−1.698	−.355	0	−.1769	.1659	.0195	−.397	4.682	−.0060	−4.6992	.03997	.301
	1.0	−32.398	−1.4005	37.8	−5.93	−.563	0	−.2701	.2186	.0843	−.490	−1.244	−.019	1.198	.093	.476
	1.8	−66.222	−2.236	70.067	−7.340	−.574	0	−.3067	.2838	.1577	−.705	−1.6167	−.0948	1.5269	.1317	.333
	.35	.943	−.314	.7824	−1.138	−.336	.1782	−.1782	0	.0106	−.350	−.01266	−.00898	.08004	.00226	.247
	1.0	−21.747	−1.2609	24.763	−3.9801	−.550	0	−.2736	.2368	.0005	−.455	−1.0078	−.0142	.9500	.0360	.289
	1.8	−24.096	−1.873	26.642	−4.963	−.784	0	−.2979	.2514	.1184	−.688	−.8434	−.0142	.7554	.1107	.215
	.35	−22.128	−.3679	23.738	−1.999	−.383	0	−.18612	.17915	.00700	.074	−.9088	−.00680	.8984	.0810	.357
	1.0	−13.715	−1.8440	18.444	−7.4601	−.778	0	−.2360	.2521	.0729	.028	−.5266	−.0228	.4498	.1177	.370
	1.8	−16.375	−3.485	18.276	8.721	.981	0	−.3911	.3473	.1720	−.414	−.4000	−.0387	.2751	.1545	.078
	.35	15.508	−.813	−14.033	−1.687	−3.066	0	−.10674	.15318	.08290	−.423	.6369	−.0048	−.6469	.0254	.372
	1.0	30.459	−1.1249	−35.096	−4.700	−.410	0	−.2433	.1819	.1142	−.549	3.435	−.016	−3.464	.076	.500
	1.8	32.402	−1.613	−27.849	−5.801	−.612	0	−.2571	.1904	.1905	−.715	.7916	−.0211	−.8409	.1082	.485

c—Due to yawing disturbance

Case	C_L	v_{N0}	v_{N1}	v_{N2}	v_{N3}	t_{vN}	p_{N0}	p_{N1}	p_{N2}	p_{N3}	t_{pN}	r_{N0}	r_{N1}	r_{N2}	r_{N3}	t_{rN}
	0.35	29.084	−0.035	−292.010	284.729	0.556	0	−0.0154	2.7364	−2.8476	0.428	1.429	−0.001	−1.948	0.644	−0.894
	1.0	90.505	.484	−232.037	142.701	.493	0	.0719	1.6813	−1.8012	.402	1.999	.007	−2.415	.600	−.997
	1.8	68.0192	2.398	−144.835	109.328	.969	0	.2300	1.149	−1.8840	.840	1.260	.040	−1.795	.519	−.269
	.35	139.162	−.026	−163.417	24.678	.078	0	−.0135	.2433	−.2430	−.130	6.836	0	−6.884	−.372	.640
	1.0	−103.736	.807	70.750	32.050	−.090	0	.0590	.4138	−.4752	.067	−2.270	.004	2.268	−.515	.934
	1.8	−184.052	1.202	151.998	31.509	−.132	0	.1631	.5074	−.6713	−.039	−3.392	.014	3.316	.557	−.965
	.35	−15.600	−.019	.6724	15.980	−.068	.1553	−.0191	0	−.1541	−.146	.0129	−.0001	.0830	−.3100	.506
	1.0	−51.788	.251	29.799	22.109	−.098	0	.0549	.3846	−.3395	−.004	−1.144	.008	1.152	−.452	.768
	1.8	−60.816	.890	38.791	21.305	−.187	0	.1414	.3660	−.5075	−.012	−1.108	.007	1.099	.491	−.857
	.35	−27.012	.073	.961	26.769	−.104	0	.0370	.00782	−.0880	.435	0	.0010	.0862	−.4202	.681
	1.0	−43.492	1.104	5.154	39.990	−.346	0	.1949	.0668	−.3948	.558	0	.0133	.1214	−.6383	.894
	1.8	−45.455	4.110	11.236	36.908	−.436	0	.4632	.1539	−.7250	.892	0	.033	.168	−.545	.835
	.35	18.980	.107	−41.150	22.326	−.064	0	−.0579	.4548	−.4372	−.170	1.894	−.002	−1.904	.858	−.603
	1.0	263.675	−.112	−310.046	26.484	−.001	0	−.0242	.6604	−.6476	−.103	12.588	−.002	−12.508	.430	−.781
	1.8	90.055	.185	−114.774	24.680	.062	0	.0215	.7555	−.8099	−.050	3.309	.002	−3.466	.437	−.711

table III, were marked off on the ordinate scale. The time intervals within which the various modes diminish or increase by one-half, or by one-tenth in the case of λ_2 (see table II), were then spaced off on the abscissa and points on the curves were found by diminishing or increasing the ordinates successively as indicated by the sign of the root. The oscillatory mode was obtained by drawing in the envelope (given by $\pm p_{Y_3}$, say) and spacing off the quarter periods, beginning at the point indicated by the phase angle of this mode. The cosine curve was then simply sketched in as shown. The final curves were found to give remarkably good checks when applied in the original differential equations of motion. Such a check shows the correctness of both the method of plotting and the analytical solutions (equation (1) and table III).

If the impressed disturbance is given as a function of t by a curve, it will usually be sufficient to approximate this curve by the addition of a number of successive positive and negative steps. The combination of steps necessary to reproduce the disturbance leads directly to the addition of the elementary motions for the resultant motion. Otherwise, for example, if the variation of disturbance is given by $L(t)$, then the resultant motion $p(t)$ at any time t due to $L(t)$ beginning at $t=0$ may be found by Duhamel's theorem, thus

$$p(t)_{\text{due to variable rolling moment}}=p_L(t)\ \ L(0)$$

$$+\int_0^t p_L(t-t_1)\ L'\ (t_1)\ dt_1 \qquad (2)$$

where $p_L(t)$ and $p_L(t-t_1)$ are obtained from table III. An explanation and a graphical method for evaluation of such integrals are given in reference 3.

MOTIONS IN SIDE GUSTS

The motion caused by a unit increment of gust velocity is found by compounding elementary disturbances in such a way as to simulate the disturbing action of the gust. Thus, in a side gust of velocity v_0, the disturbing acceleration along Y will be v_0Y_v and angular disturbances will be v_0L_v, v_0N_v.

As explained before, the effects of any usual variation of gustiness can be largely foretold from the effect of a unit sharp-edge gust. The variable gust can be built up from small increment jumps of gust velocity corresponding to sharp-edge cross-currents and the final motion will approach that obtained by superposing the motions due to the individual elements.

The effect of a sharp gust from the side is similar to, although not exactly the same as, the effect of an initial angle of sideslip. For the side gust, it is necessary to take account of the period of penetration of the airplane into the current. The first effect will be to push the nose of the airplane downwind whereas an instant later the current will strike the fin, turning the machine into the gust. The action of dihedral in causing the machine to roll away from the gust will also occur before the fin is affected. These effects are, however, of short duration and do not alter the motion to any great extent after the first fraction of a second, except in cases of small weathercock stability where the fuselage contributes a large unstable yawing moment.

The computations that follow are based on the assumption that the rolling action of the gust begins at $t=0$ and that the yawing action begins when the airplane has traveled far enough to carry the fin into the current. The case of $C_{n_\beta}=0$ was treated by assuming a yawing couple equal and opposite to that of the 4-percent fin applied when $t=0$, this couple being neutralized at the instant the fin entered the gust.

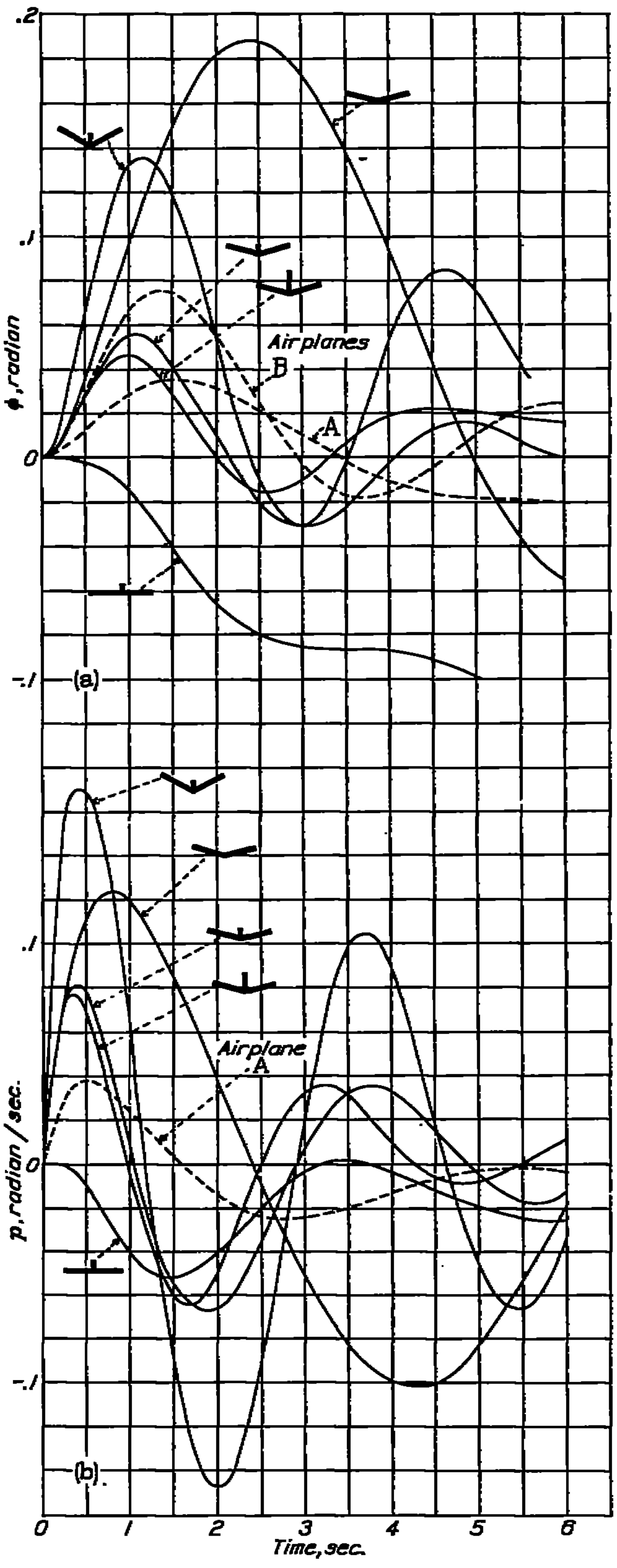
A possible further refinement of the calculations would involve the delay in building up the full lift forces on the various surfaces. Mathematical methods for dealing with various lags or rates of growth of the aerodynamic reactions have been developed, but their description is beyond the intended scope of the present report. It may be said, however, that, for motions as slow as the natural oscillations of a rigid airplane, this effect (judging by the theoretical predictions) is quite negligible.

Figure 3 illustrates the results of the calculations based on a 10-foot-per-second sharp-edge side gust. The curves shown are for flight at $C_L=1.0$ but the same general trends appeared in the calculations made for other lift coefficients.

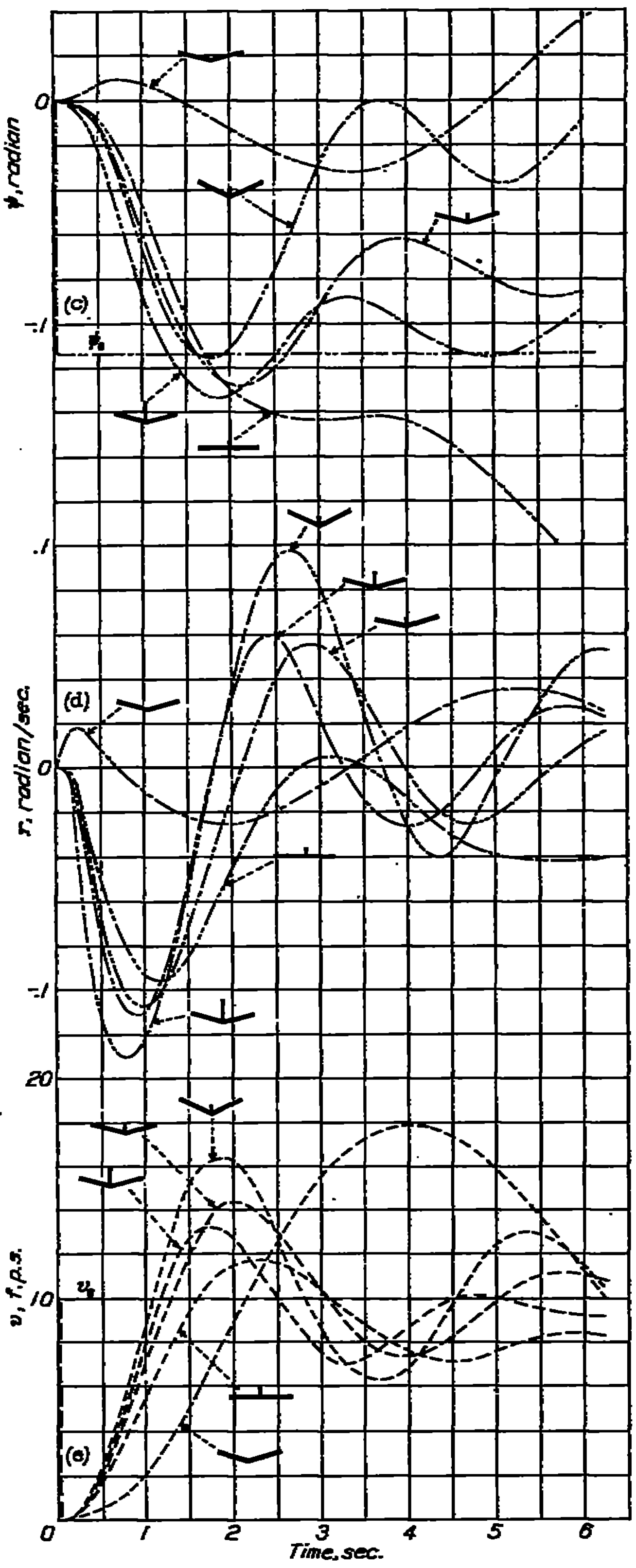
The most noteworthy difference shown is the effect of deficiency of fin area on the banking motion (figs. 3 (a) and 3 (b)). The airplane with 10° dihedral and average fin was not displaced so much in bank by the side gust as was the airplane with 5° dihedral and a small fin. The initial rate of rolling, however, was greater with the greater dihedral.

With a given dihedral an increase of fin area cuts down the banking motion although, after a certain size is reached, the gain becomes slight, as is illustrated by figure 3 (a). With neutral weathercock stability (fig. 3 (c)) the change of heading on entering the gust is at first small and, although the motion is stable, the oscillation in azimuth seems to be reinforced for a time by the rolling. This action is to be attributed to the phase lag between the rolling and the yawing effects upon penetrating the sharp-edge current. It seems probable that an appearance of inherent instability may be reached at a point considerably above the mathematical limit for undamped oscillations. (See fig. 1.) It is known, for instance, that unstable oscillations may result from an attempt to hold the wings level with ordinary ailerons unless C_{n_β} has a definite positive value.

The side gust is equivalent to a sudden shift in the wind direction, corresponding to a change in azimuth ψ_0 as indicated in figure 3 (c). The normal airplane swings about and tends to approach this heading. It will be noted that the airplane with the large fin turns fairly sharply into the wind and, since the banking



(a) Banking.
(b) Rolling motion.



(c) Azimuth.
(d) Yawing motion.
(e) Sideslip velocity.

FIGURE 3.—Motions caused by a sharp-edge side gust.
C_L=1.0; v_g=10 f. p. s.; U_0=88.5 f. p. s.

motion is small, tends to keep the same flight path relative to the earth for a short time. After about 6 seconds, however, the spiral divergence begins to be apparent and the motion finally results in turning downwind.

An example of extreme spiral divergence is illustrated by the airplane with no dihedral. In this case, however, the airplane banks and turns directly upwind. The airplane with large dihedral illustrates the opposite condition and shows the predominance of oscillations that generally characterizes the effect of dihedral. Here the airplane tends back toward its original azimuth heading, drifting sidewise with the gust.

The airplane with 5° dihedral banked rather sharply away from the gust, whereas the airplane with zero dihedral showed an undesirable tendency to bank and slide into the gust. It was therefore a matter of interest to try some modifications lying in between these two conditions. It was realized that the rolling could not be entirely suppressed by such modifications on account of the phase relationships involved in the motions.

It appeared that 1° or 2° of effective dihedral would give about the least banking motion in the side gust and hence this condition was investigated. Inasmuch as the airplane might have shown a noticeable spiral divergence at low speeds with the normal fin area, this area was arbitrarily reduced, bringing the weathercock-stability factor C_{n_β} down in about the same proportion as the dihedral factor C_{l_β}. The values selected were $C_{n_\beta}=0.025$ and $C_{l_\beta}=-0.035$, which corresponds to 2° effective dihedral. The position of this airplane on the lateral-stability chart is denoted by the point A in figure 1 (b).

The results for airplane A are compared with the others in figures 3 (a) and 3 (b). It will be noted that the bank is somewhat smaller than in the case with 5° dihedral and a large fin but that the bank persists for a longer time. The difference made by the change from 5° dihedral to 2° seems surprisingly small. A somewhat greater difference would be expected if the fin had not been reduced. It should be borne in mind that the yawing disturbance is reduced by cutting down the fin.

The curve for airplane B (fig. 3 (a)) shows the result of attempting to secure spiral stability (at $C_L=1.0$) by cutting down the fin of the airplane with 5° dihedral. (Note that airplane A is slightly unstable.) The value of C_{n_β} in this case is about half that assumed for the mean condition. (See fig. 1 (b).) The banking displacement seems undesirably large (comparatively) in this case.

OTHER TYPES OF GUST

The flight velocity of the airplane being normally large with respect to gust velocities, it is permissible to consider the gusts as being stationary in time with respect to the flight path. Thus the gusts are considered to exist as a fixed pattern in the air ahead of the airplane and not to vary in time within the short space required for the machine to travel its own length.

As mentioned before, when the airplane enters a cross-current in level flight, a gradient of sidewise velocity along the length of the fuselage will exist. The effect of this gradient is similar to the effect of a relative yawing motion superposed on the side velocity. For a uniform gradient the additional yawing moment would be $(-dv_0/dx)\times N_r$. The calculations involved this factor by virtue of the time lag assumed in application of the yawing moment due to the fin, and upon this basis they should be applicable to any reasonable variation or gradient of sidewise velocity.

A somewhat different situation arises when the airplane is climbing or descending through a cross wind that varies with height, as, for instance, when descending through the earth boundary layer for a cross-wind landing, for then no perceptible gradient of sidewise velocity along the length of the airplane will exist. The motions that arise in these cases can be compounded by integration from the motion following an initial angle of sideslip. This motion is not greatly different from that caused by entering a sharp cross-current and the same general conclusions will apply.

It appears that a true yawing gust, consisting of pure angular relative motion of the air, could act only momentarily on the airplane. The sidewise velocity would predominate after the first two- or three-tenths of a second with the airplane flying at normal speed. Gradients of velocity along the wing span, however, might persist for longer periods.

Away from the ground influence, gradients of forward velocity and of vertical velocity along the wing span must be considered as being about equally probable. At normal flight speeds, the vertical gradients produce by far the greater effects. As was mentioned before, the damping of relative rolling motion is such that the airplane very quickly takes on the angular velocity of the gust gradient.

Figure 4 shows the rolling motion calculated for the medium airplane (5° dihedral and 6 percent fin) in a momentary rolling gust $p_0b/2U_0=0.05$. It will be noted that the airplane takes on approximately half the rolling velocity of the gust within one-fifth second.

As might be expected, observations have shown that vertical currents tend to diminish near flat ground. Thus side gusts and yawing gradients are more likely to affect the airplane while it is landing and taking off. At very low speeds with flaps down, the rolling derivative due to yawing becomes as great as that due to rolling (see table I) so that in this condition the airplane is affected as much by spanwise gradients of longitudinal velocity as it would be by the rolling gradients. (Note also that the effect of the rolling gradients is less at low speed.)

Figure 5 shows the banking reactions of the various airplanes in a sudden, persistent yawing-gradient gust.

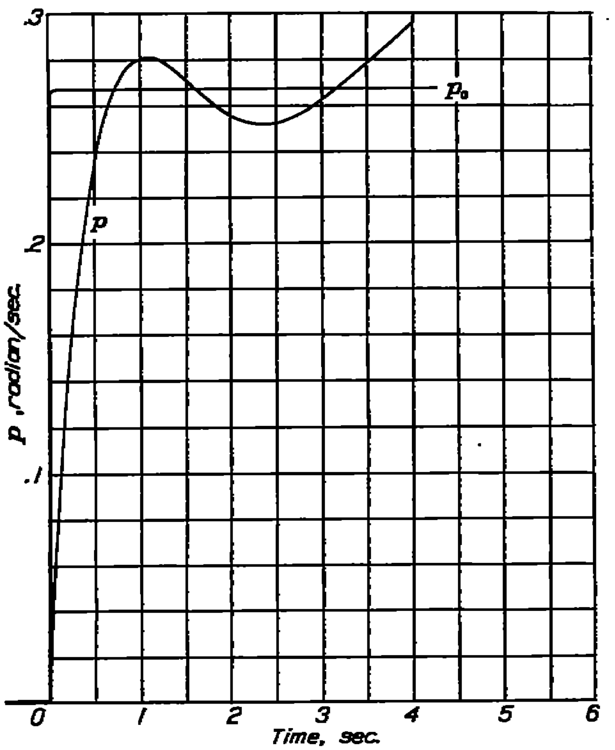


FIGURE 4.—Typical rolling motion caused by a sudden rolling gradient.

$\frac{p_0 b}{2U_0}=0.05(t>0)$; $U_0=88.5$ f. p. s.

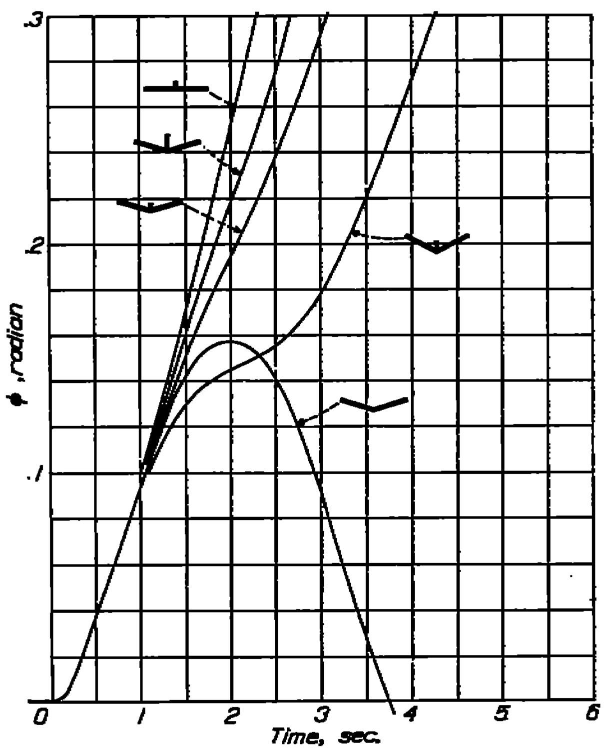


FIGURE 5.—Bank caused by a yawing gradient along the wing span.

$\frac{r_0 b}{2U_0}=0.05(t>0)$; $U_0=88.5$ f. p. s.

The gust assumed corresponded to a difference of longitudinal velocity between the two wing tips of about 9 feet per second ($r_0 b/2U_0=0.05$). All the examples show roughly the same banking tendency within the first second, since the disturbing factors (rolling moments due to yawing velocity) are the same in all cases. The subsequent motions show the influence of different degrees of spiral stability and instability.

The primary disturbing factor in the yawing-gradient gust being proportional to the derivative L_r (rolling moment due to yawing), the greatest room for improvement would be to reduce this derivative. Taper and washout (such as are attained with a partial-span flap) are beneficial in this respect. It is estimated that L_r might have been reduced by one-third of the given value (see table I) at $C_L=1.8$ if a 50-percent-span flap had been assumed. The effect of plan form is not so pronounced, leading to a reduction of one-sixth for a 4:1 taper.

In general, a reduction of the rolling moment due to yawing seems desirable from considerations of lateral stability. The magnitude of this derivative with controls fixed is, like that due to rolling, primarily a consequence of the general lay-out of the airplane and is not dictated by considerations of stability. It appears, however, that the magnitude of L_r with controls free (or loosely held) could be reduced or reversed by making use of an appropriate combination of ailerons with increased upward pressure (attained by cambering the ailerons) and a differential linkage, as described in reference 10. An appropriate linkage would eliminate the necessity of applying contrary aileron pressure during steady turns and would also eliminate the spiral instability with controls free.

CONCLUDING REMARKS

A study of the effects of gusts gives different indications depending on the interval of time considered. During the first stages, the upsetting movements of the stable airplane may be more severe than those of a slightly unstable one. If the airplane is under control and if the gusts are of noticeable magnitude, then the motion during the first 2 or 3 seconds is of primary concern. For uncontrolled flight or for flight in relatively calm air where disturbances could become apparent only through introducing a divergence, the later stages of the motion are of interest.

In a consideration of the early stages of the motion, it is evident that the requirements of fin area and dihedral for spiral stability at low speed conflict somewhat with the requirements for steadiness in side gusts. If spiral instability is present, the rates of divergence introduced by various disturbances appear to be small as long as there is a moderate dihedral action present. The condition of zero (effective) dihedral leads, however, to definitely undesirable rates of divergence.

If average weathercock stability ($C_{n_\beta}=0.05$ to 0.07) is assumed, the optimum magnitude of the rolling derivative due to sideslip for steadiness in side gusts appears to be about $C_{l_\beta}=-0.01$ to -0.04, corresponding to an effective dihedral of 1° or 2°. Spiral stability throughout the flight range could be secured with this dihedral by cutting down the fin effect. The latter change would lead to somewhat greater banking displacements in the gusts and would also be detrimental to aileron control, unless such control were obtained without adverse yawing moments.

The damping of rolling is such that the airplane very quickly takes on any rolling component of gust velocity. The usual modifications of the lateral-stability factors have but little influence on the immediate effects of the rolling gust. An automatic device, acting so as to cut down the damping of rolling (relative to the air), should be advantageous from considerations of riding comfort.

The effects of longitudinal gradients of gust velocity become fairly large at low flight speeds. Noticeable improvement can be obtained by the use of partial-span flaps or by otherwise concentrating the lift toward the center of the wing, but this conclusion applies, of course, only as long as no portion of the wing is brought near the stalling point.

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National Advisory Committee for Aeronautics,
Langley Field, Va., *June 8, 1938.*

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